J10 CHAP 4, pps 173-196

P 174

[1] $f(x) = x^2 - 2x + 3$

[1,1] $f(1) = 2$

[1,2] $f(0) = 3$

[1,3] $f(-3) = 18$

[1,4] $f(a) = a^2 - 2a + 3$

P 175

[2,1] RNG = $\mathbb{R}$

[2,2] RNG = $\{y \ni y \geq -1\}$

[2,3] RNG = $\{y \ni y \geq 2\}$

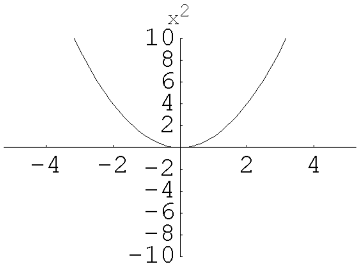
[3] RNG = $\{y \ni y > 0\}$

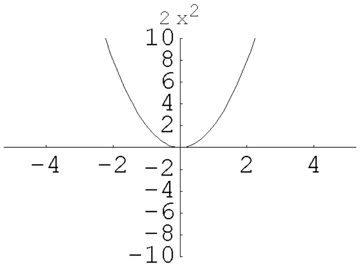
[4,1] $\{y \ni y > 3\}$

[4,2] $\{y \ni y \geq 0\}$

p 177

[1]


x^2
10
8
6
4
2
-4
-2
2
4
-2
-4
-6
-8
-10


$2 x^2$
10
8
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4
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-2
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4
-2
-4
-6
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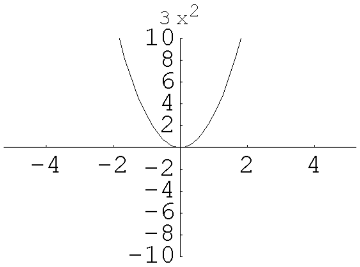
$3 x^2$
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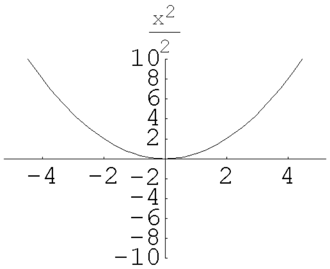
$\frac{x^2}{2}$
10
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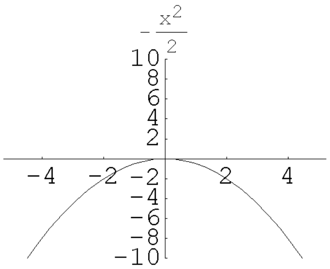
$-\frac{x^2}{2}$
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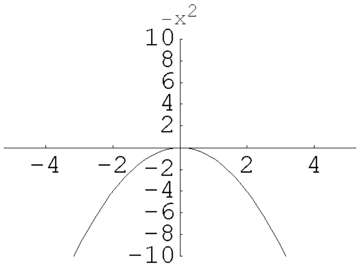
$-x^2$
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-4
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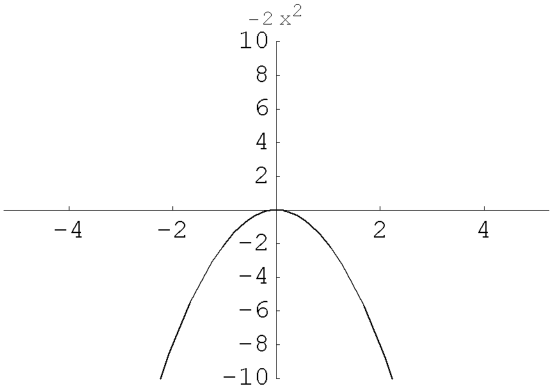
$-2 x^2$
10
8
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4
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-4
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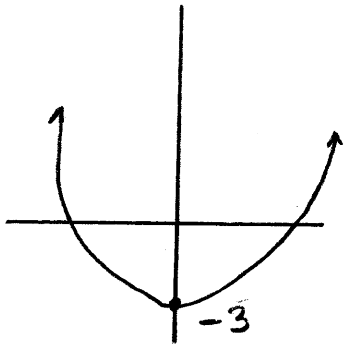
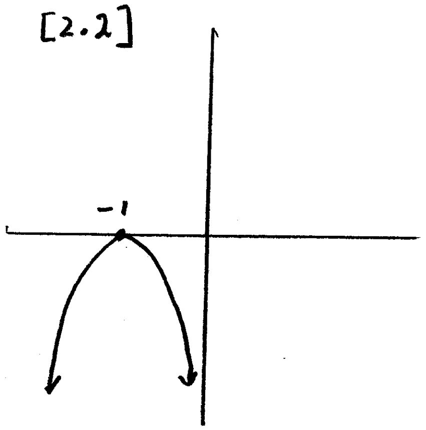
2
4
-2
-4
-6
-8
-10

P179

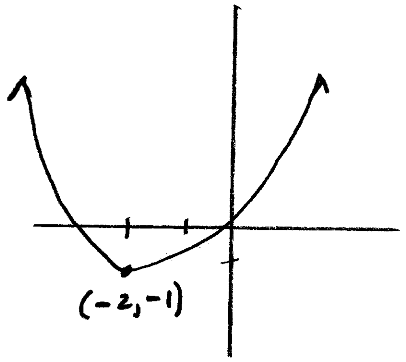
[2.1]



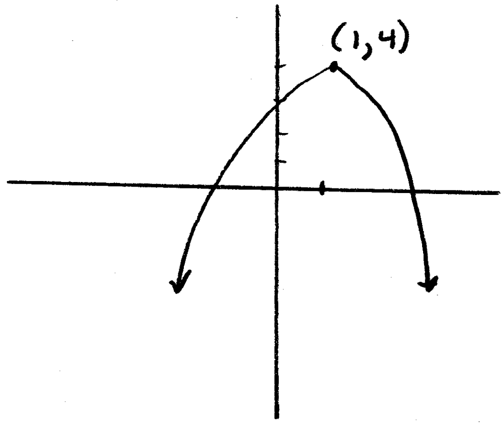
[2.2]



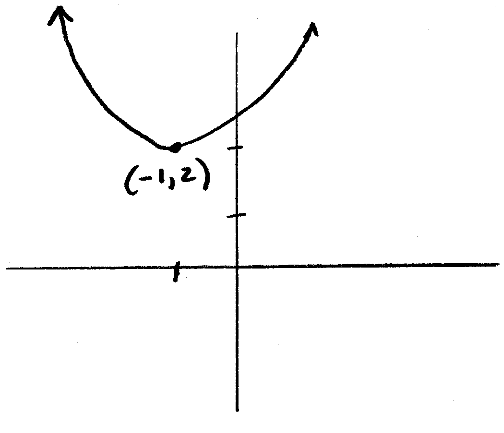
[2.3]



[2.4]



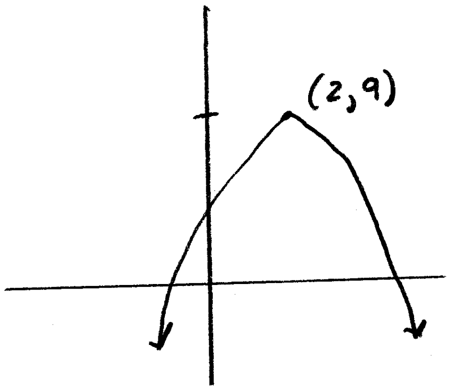
[3.1] $y = (x+1)^2 + 2$



[3.2] $y = -2(x^2 - 4x) + 1$

$= -2(x-2)^2 + 1 + 8$

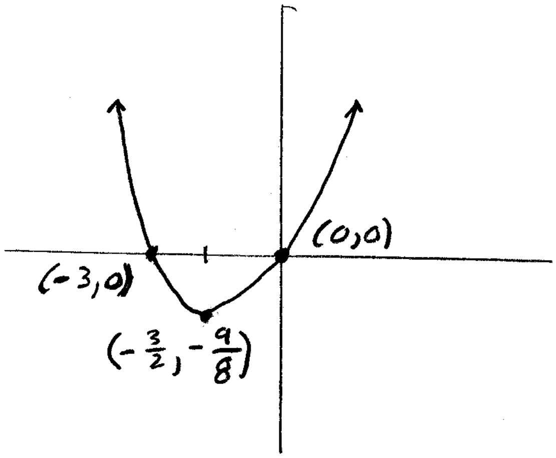
$= -2(x-2)^2 + 9$



P179, ctd

[3.3] $y = 2x^2 + 3x$

$= 2(x^2 + \frac{3}{2})^2 - \frac{9}{8}$



P180

[4]

$$\left.\begin{aligned} y &= ax^2 + bx + c \\ &= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - \frac{b^2}{4a} \\ &= a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac}{4a} - \frac{b^2}{4a} \\ &= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a} \end{aligned}\right]$$

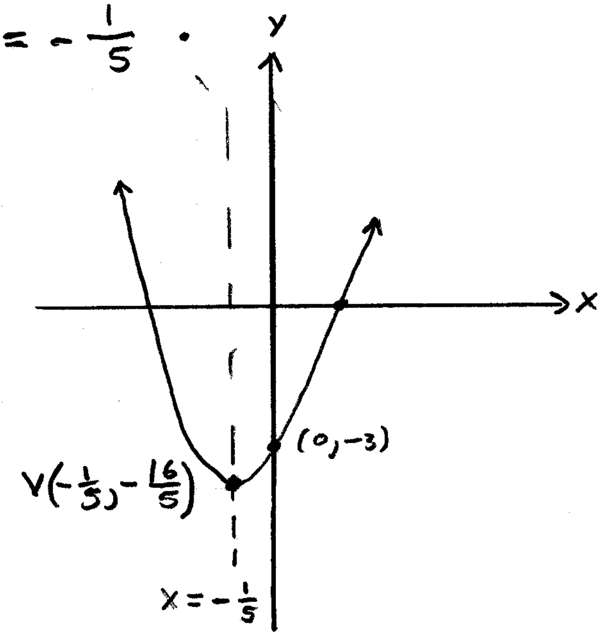
<u>Note</u>

This is the beginning of deriving the Quadratic formula.

[5.1] $y = 5x^2 + 2x - 3$

$V\left(-\frac{2}{10}, -\frac{4+60}{20}\right) = \left(-\frac{1}{5}, -\frac{16}{5}\right)$

axis of symmetry: $x = -\frac{1}{5}$.

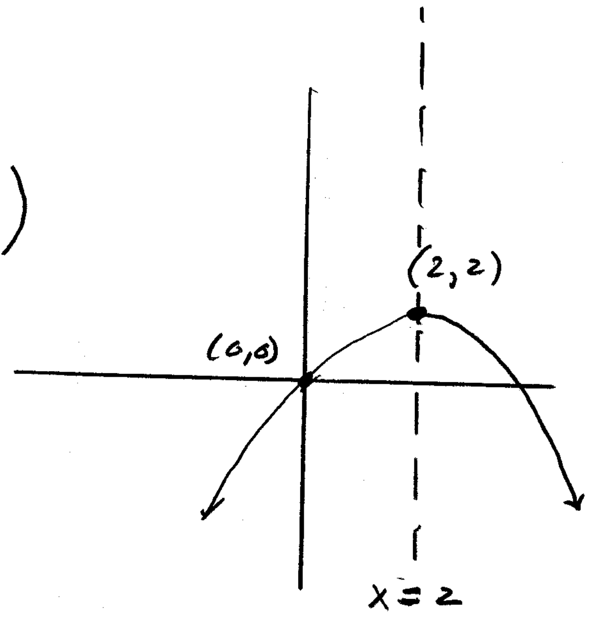


P180, ctd

[5.2] $y = -\frac{1}{2}x^2 + 2x$

$$v\left(-\frac{2}{-1}, -\frac{4-0}{-2}\right) = (2, 2)$$

axis: $x = 2$



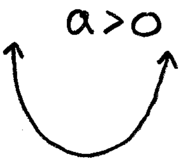
NOTE: It is a lot easier to complete the square than to rememeber and use this formula. Forget the formula. Do all such problems by completing the square.

P182

[6.1] $y = x^2 - 6x + 2$

$y = (x-3)^2 + 2 - 9$

$= (x-3)^2 - 7$



$\therefore$ The minimum value of the function is -7 and it takes this value when $x = 3$.

→

P 182, ctd

[6.2]

$$y = \frac{1}{2}x^2 + x$$

$$= \frac{1}{2}(x^2 + 2x)$$

$$= \frac{1}{2}(x^2 + 2x + 1) - \frac{1}{2}$$

$$= \frac{1}{2}(x+1)^2 - \frac{1}{2}$$

$\therefore$ min is $-\frac{1}{2}$ at $x = -1$.

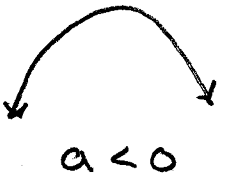
[6.3]

$$y = -2x^2 + 3x$$

$$y = -2\left(x^2 - \frac{3}{2}x\right)$$

$$= -2\left(x^2 - \frac{3}{2}x + \frac{9}{16}\right) + \frac{9}{8}$$

$$= -2\left(x - \frac{3}{4}\right)^2 + \frac{9}{8}$$

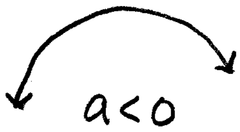


$\therefore$ MAX value of $\frac{9}{8}$ at $x = \frac{3}{4}$

→

P182, ctd

[6.4] $y = 4 - 2x - \frac{1}{2}x^2$



$= -\frac{1}{2}x^2 - 2x + 4$

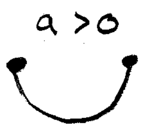
$= -\frac{1}{2}(x^2 + 4x) + 4$

$= -\frac{1}{2}(x^2 + 4x + 4) + 4 + 2$

$= -\frac{1}{2}(x+2)^2 + 6$

$\therefore$ max value is 6 at $x = -2$

<u>P 183</u>



[7.1] $y = x^2 - 4x, \quad 0 \leq x \leq 3$

$= (x-2)^2 - 4$

so $\boxed{\text{min is } -4 \text{ at } x = 2}$

Now find max value

$f(0) = 0 - 0 = 0$

$f(3) = 9 - 12 = -3$

so $\boxed{\text{max value is } 0 \text{ at } x = 0}$

→

P183, ctd

[7.2] $y = x^2 + x - 2, \quad -2 \le x \le 1$

$= (x + \frac{1}{2})^2 - 2 - \frac{1}{4}$

$= (x - \frac{1}{2})^2 - \frac{9}{4}$

$\boxed{\text{min is } -\frac{9}{4} \text{ at } x = \frac{1}{2}}$

Find max

$f(-2) = 4 - 2 - 2 = 0$

$f(1) = 1 + 1 - 2 = 0$

$\boxed{\text{max is zero at } x = -2 \text{ and } x = 1}$

[7.3] $y = -x^2 + 5, \quad -1 \le x \le 2$

$= -(x + 0)^2 + 5 + 0$

$\boxed{\text{max is } 5 \text{ at } x = 0}$

$f(-1) = -1 + 5 = 4$

$f(2) = -4 + 5 = 1$

$\boxed{\text{min is } 1 \text{ at } x = 2}$

P 183, ctd

[7.4] $y = -x^2 + 3x, \quad -2 \le x \le 1$

$a<0$

$$= -(x^2 - 3x)$$

$$= -\left(x^2 - 3x + \frac{9}{4}\right) + \frac{9}{4}$$

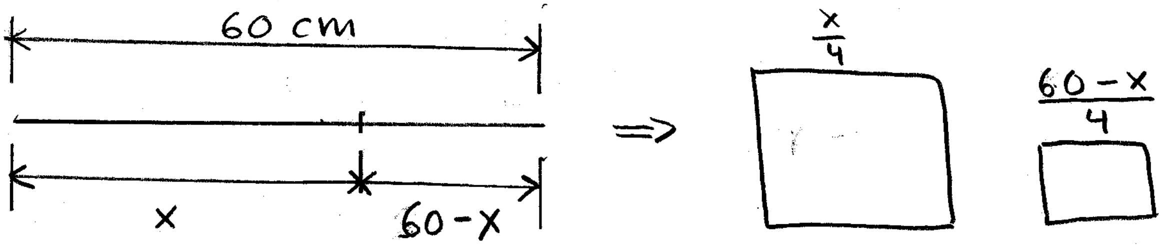
$$= -\left(x - \frac{3}{2}\right)^2 + \frac{9}{4}$$

max is $\frac{9}{4}$ at $x = \frac{3}{2}$

$f(-2) = -4 - 6 = -10$
$f(1) = -1 + 3 = 2$

min is -10 at $x = -2$

[8]



$$A(x) = \left[\frac{x}{4}\right]^2 + \left[\frac{60-x}{4}\right]^2$$

$$= \frac{1}{16}\left[x^2 + x^2 - 120x + 60^2\right]$$

$$= \frac{1}{8}\left[x^2 - 60x + (30)(60)\right]$$

$$= \frac{1}{8}\left[(x-30)^2 + \right.$$ stuff you do <u>not</u> care about because you need only to find x at which the min occurs; you are not asked what the minimum value is.]

∴ min when $x = 30$
so cut wire in half.

P185

[1] part (i) no modification
(ii) no modification
(iii) no modification

But the graphs will be re drawn concave down.

[2] $y = x^2 - kx + 4$

$D = k^2 - 16$

$D > 0 \Leftrightarrow k > 4$ Two real solns (Two intersections)
$D = 0 \Leftrightarrow k = 4$ one real soln, multiplicity 2 (tangent)
$D < 0 \Leftrightarrow k < 4$ no real solns. (no intersections)

P 186

[3.1] $\begin{bmatrix} y = x^2 + 3 \\ y = -2x + n \end{bmatrix}$

$$\equiv \quad x^2 + 3 = -2x + n$$

$$x^2 + 2x + 3 - n = 0$$

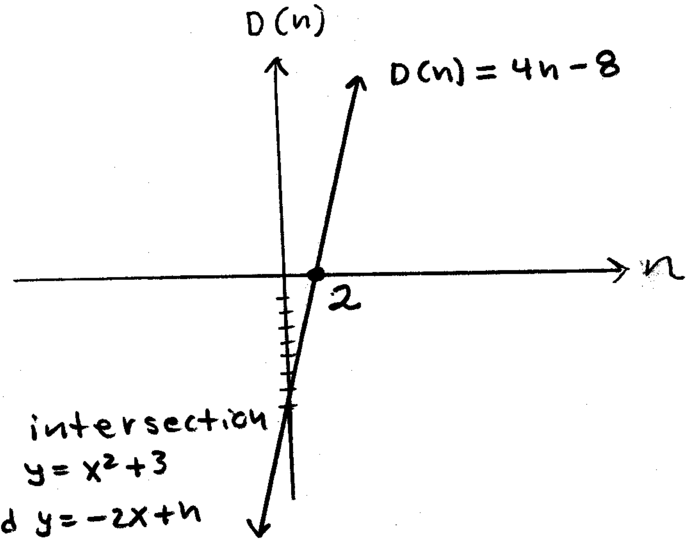
The discriminant is a function of n.

$$D(n) = 4 - 4(3 - n)$$

$$= 4 - 12 + 4n$$

$$D(n) = 4n - 8$$

We wish to know when $D(n)$ is pos, zero, or negative.



∴

$n < 2 \Rightarrow D(n) < 0 \Rightarrow$ no intersection of $y = x^2 + 3$ and $y = -2x + n$

$n = 2 \Rightarrow D(n) = 0 \Rightarrow y = x^2 + 3$ and $y = -2x + n$ intersect at on point

$n > 2 \Rightarrow D(n) > 0 \Rightarrow y = x^2 + 3$ and $y = -2x + n$ intersect at two points.

P186, ctd

[3.2] $$\begin{bmatrix} y = x^2+3 \\ y = mx-2 \end{bmatrix}$$

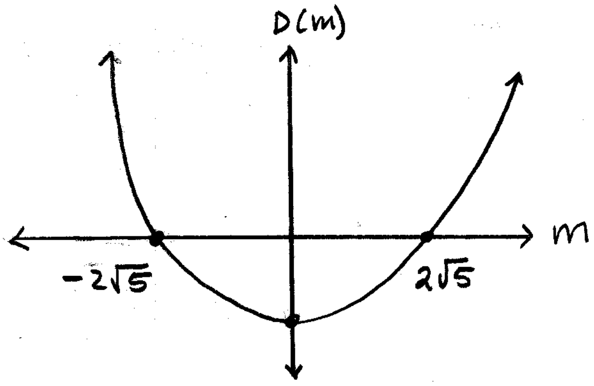
$$\equiv \quad x^2+3 = mx-2$$

$$x^2 - mx + 5 = 0$$

Now the discriminant is a function of m; that is,

$$D(m) = m^2 - 20.$$

we wish to knw when D(m) is Negative, zero, or positive.



$\therefore$ $-2\sqrt{5} < m < 2\sqrt{5} \Rightarrow D(m) < 0 \Rightarrow$ no intersections of $y = x^2+3$ and $y = mx-2$

$m = \pm 2\sqrt{5} \Rightarrow$ $y = x^2+3$ and $y = mx-2$ intersect at a single point.

$m < -2\sqrt{5}$ or $m > 2\sqrt{5} \Rightarrow$ $y = x^2+3$ and $y = mx-2$ intersect at two points.

P188

[4,1] $x^2 + 4x + 1 < 0$.

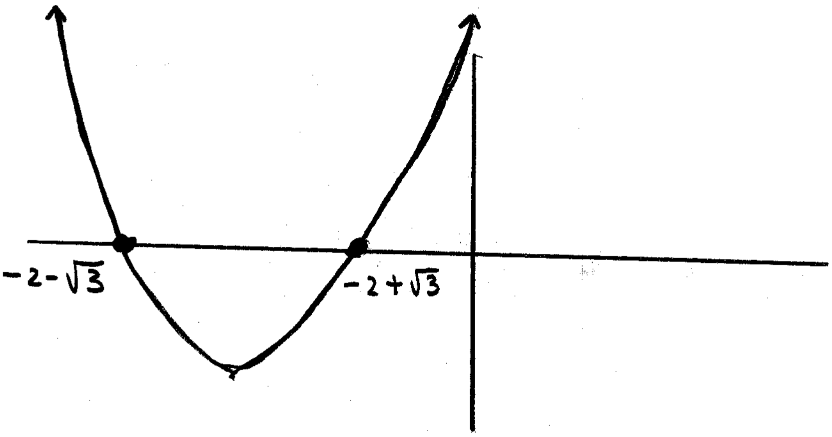
Consider $f(x) = x^2 + 4x + 1$. To find where this function crosses x-axis, set $f(x) = 0$ and Solve for x.

$$x^2 + 4x + 1 = 0$$

$$x = \frac{-4 \pm \sqrt{16 - 4}}{2}$$

$$= \frac{-4 \pm 2\sqrt{3}}{2}$$

$$= -2 \pm \sqrt{3}$$



$\therefore$ $x^2 + 4x + 1 < 0$ for $-2-\sqrt{3} < x < -2+\sqrt{3}$

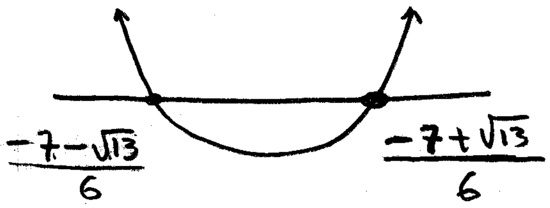
p 188, ctd

[4.2] $3x^2 + 7x + 3 \geqslant 0$

$f(x) = 3x^2 + 7x + 3$

$f(x) = 0 \Rightarrow x = \frac{-7 \pm \sqrt{49-36}}{6} \Rightarrow x = -\frac{7+\sqrt{13}}{6}, \; x = -\frac{7-\sqrt{13}}{6}$

graph concave up.



$\therefore$ $3x^2+7x+3 \geqslant 0$ when $x \leqslant \frac{-7-\sqrt{13}}{6}$ or $x \geqslant \frac{-7+\sqrt{13}}{6}$.

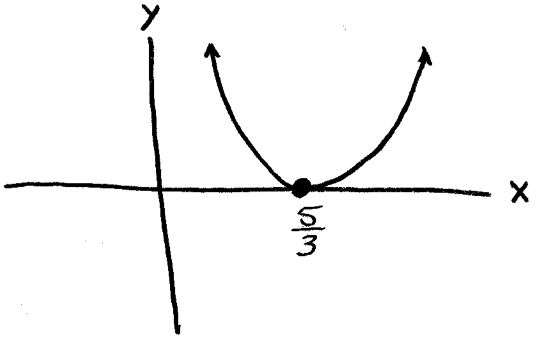
or you could write

$$x \in \left[\frac{-7-\sqrt{13}}{6}, \frac{-7+\sqrt{13}}{6}\right]$$

[4.3] $9x^2 - 30x + 25 > 0$

$f(x) = 9x^2 - 30x + 25$

$f(x) = 0 \Rightarrow x = \frac{5}{3}$



$\therefore$ $x < \frac{5}{3}$ or $x > \frac{5}{3}$

or write $x \in (-\infty, \frac{5}{3}) \cup (\frac{5}{3}, \infty+)$

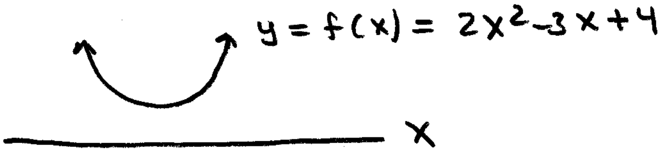
or write $x \in \mathbb{R} - \{\frac{5}{3}\}$

P 188, ctd

[4.4] $2x^2-3x+4<0$

$f(x) = 2x^2-3x+4$

$f(x)=0 \Rightarrow D = 9-(4)(2)(4) < 0$, so graph of $f(x)=2x^2-3x+4$ lies entirely above the x-axis (since $a=2>0$).

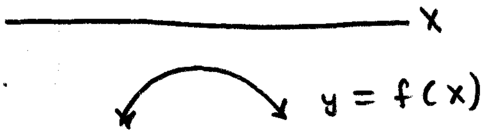


$\therefore$ $x \in \{\}$ or there is no $x \in \mathbb{R}$ for which $2x^2-3x+4<0$.

[4.5] $-x^2+5x-7<0$

$f(x) = -x^2+5x-7$.

$f(x)=0 \Rightarrow D = 25 - 4(-1)(-7) = 25-28 < 0$, so graph of $f(x)$ is entirely below x-axis



$\therefore$ $x \in \mathbb{R}$. Every real number makes $-x^2+5x-7<0$ true

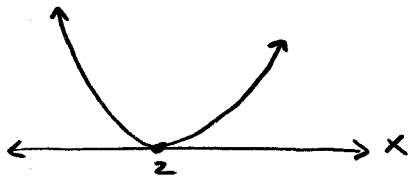
→

P188, ctd

[4.6] $x^2 - 4x + 4 \leq 0$

$f(x) = x^2 - 4x + 4$

$f(x) = 0 \Rightarrow x^2 - 4x + 4 = 0 \Rightarrow x = 2$



$\therefore \quad x = 2$

[5] Skip

P189

[6] Skip for now

P191

[1.1] $y = -3(x+2)^2$

$= -3(x^2+4x+4]$

$y = -3x^2-12x-12$

[1.2] $y = -3(X-0)^2-3$

$y = -3X^2-3$

[1.3] $y = -3(x-5)^2+7$

$= -3[x^2-10x+25]+7$

$= -3x^2+10x-75+7$

$= -3x^2+10X-68$

[1.4] $y = -3(x+1)^2+4$

$= -3(x^2+2x+1)+4$

$= -3x^2-6X-3+4$

$= -3X^2-6X+1$

P 191, ctd

[2.1] $y = 3x^2 - 2x$

Find y-intercept.

Set $x = 0 \Rightarrow y = 0$

So $\boxed{(0,0) \text{ on graph}}$

Find x-intercepts

set $y = 0$

$$3x^2 - 2x = 0$$

$$x(3x - 2) = 0$$

$$x = 0 \quad \text{or} \quad x = \frac{2}{3}$$

So $\boxed{(0,0),\ \left(\frac{2}{3}, 0\right) \text{ on Graph}}$

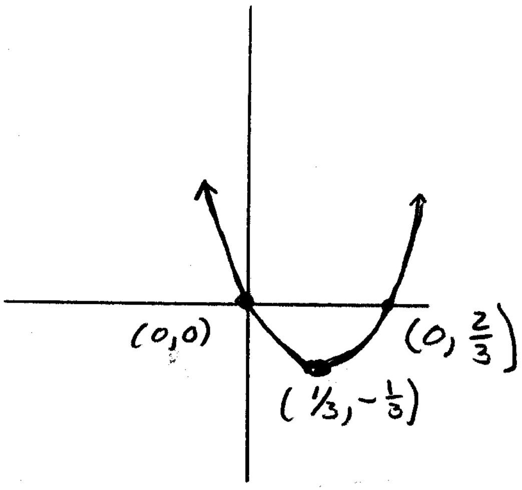
Vertex: $x = \frac{1}{2}\left(0 + \frac{2}{3}\right) = \frac{2}{6} = \frac{1}{3}$

$$f\left(\tfrac{1}{3}\right) = 3\left[\tfrac{1}{3}\right]^2 - 2\left[\tfrac{1}{3}\right]$$

$$= \frac{1}{3} - \frac{2}{3}$$

$$= -\frac{1}{3}$$

So $\boxed{V\left(\frac{1}{3}, -\frac{1}{3}\right)}$



P191, c+d

[2,2] $y = (x+1)(x-3)$

$$\boxed{(-1,0),\ (3,0) \in F}$$

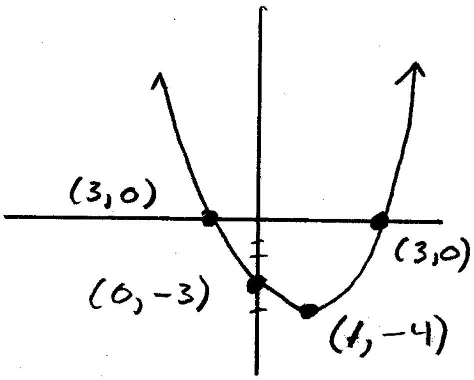
$x=0 \Rightarrow y = (1)(-3)$

$$\boxed{(0,-3) \in F}$$

Vertex $V_x = \frac{-1+3}{2} = 1$

$f(1) = 2(-2) = -4$

$$\boxed{V(1,-4)}$$



P 191, ctd

[2,3] $y = -2x^2 - 4x + 6$

$x = 0 \Rightarrow y = 6 \Rightarrow$ $\boxed{(0,6) \in F}$

$y = 0 \Rightarrow$

$$-2x^2 - 4x + 6 = 0$$

$$2x^2 + 4x - 6 = 0$$

$$x^2 + 2x - 3 = 0$$

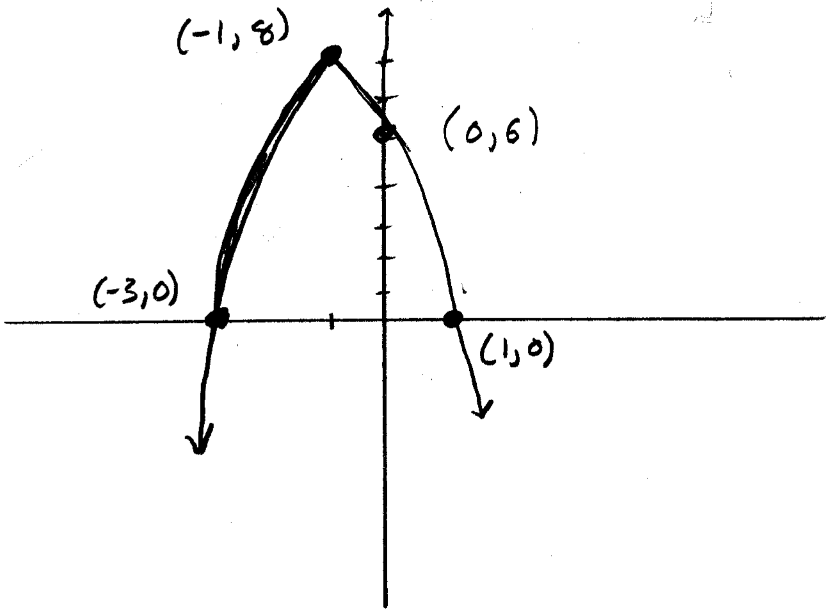
$$(x+3)(x-1) = 0$$

$\boxed{(-3,0), (1,0) \in F}$

$V_x = \frac{1}{2}(-3+1) = \frac{1}{2}(-2) = -1$

$f(-1) = -2 + 4 + 6 = 8$

$\boxed{V(-1, 8)}$



P 191, ctd

[3.1] $y = a(x-h)^2 + k$

$y = a(x-3)^2 - 1$

$0 = a(6-3)^2 - 1$

$0 = 9a - 1$

$a = \frac{1}{9}$

$\therefore \quad y = \frac{1}{9}(x-3) - 1$

[3.2] $y = ax^2 + bx + c$

$$\left[\begin{array}{l} a - b + c = -3 \\ \quad\quad\; c = 1 \\ a + b + c = 3 \end{array}\right] \equiv \left[\begin{array}{l} a - b = -4 \\ a + b = 2 \\ \end{array}\right] \equiv 2a = -2 \Rightarrow a = -1$$

$a = -1 \Rightarrow -1 + b + 1 = 3 \Rightarrow b = 3$

$\therefore \quad y = -x^2 + 3x + 1$

[3.3] $V(x, 0)$ and $(1,1), (4,4)$

P 191

[3.3] Since the vertex is at $(h, 0)$, Eqn parabola is

$$y = a(x-h)^2$$

$$\equiv \quad y = a(x^2 - 2hx + h^2)$$

$$\equiv \quad x^2 - 2hx + h^2 = \frac{y}{a}$$

Since points $(1,1)$ and $(4,4)$ on parabola,

$$\left[\begin{array}{l} 1 - 2h + h^2 = \frac{1}{a} \\ 16 - 8h + h^2 = \frac{4}{a} \end{array}\right] \equiv \left[\begin{array}{l} 4 - 8h + 4h^2 = \frac{4}{a} \\ 16 - 8h + h^2 = \frac{4}{a} \end{array}\right] \Rightarrow \boxed{h = \pm 2}$$

$h = 2$

$1 - 4 + 4 = \frac{1}{a}$

$a = 1$

<u>or</u>

$h = -2$

$1 + 4 + 4 = \frac{1}{a}$

$a = \frac{1}{9}$

$$\boxed{\therefore \quad y = (x-2)^2 \quad \underline{\text{or}} \quad y = \frac{1}{9}(x+2)^2}$$

Both equations should be checked for $(1,1)$ and $(4,4)$.

$1 = (1-2)^2$ ✓

$4 = (4-2)^2$ ✓

$1 = \frac{1}{9}(1+3)^2$ ✓

$4 = \frac{1}{9}(4-2)$ ✓

[4] Skip

P192

[5.1] $y = 3x - x^2 \quad , \quad -1 \le x \le 2$

$$y = -(x^2 - 3x)$$

$$= -(x^2 - 3x + \tfrac{9}{4}) + \tfrac{9}{4}$$

$$= -(x - \tfrac{3}{2})^2 + \tfrac{9}{4}$$

$$\boxed{\therefore \text{ max is } \tfrac{9}{4} \text{ at } x = \tfrac{3}{2}}$$

$$f(-1) = 3(-1) - (-1)^2 = -3 - 1 = -4$$

$$f(2) = 3(2) - 2^2 = 6 - 4 = 2$$

$$\boxed{\therefore \text{ min is } -4 \text{ at } x = -1}$$

[5.2] $y = x^2 + 5x + 4 \quad , \quad -3 \le x \le 0$

$$y = (x^2 + 5x + \tfrac{25}{4}) + 4 - \tfrac{25}{4}$$

$$= (x + \tfrac{5}{2})^2 - \tfrac{9}{4}$$

$$\boxed{\therefore \text{ min is } -\tfrac{9}{4} \text{ when } x = -\tfrac{5}{2}}$$

$$f(-3) = 9 - 15 + 4 = -2$$

$$f(0) = 0 + 0 + 4 = 4$$

$$\boxed{\therefore \text{ max is } 4 \text{ at } x = 0}$$

p 192

[6] $\begin{bmatrix} y = x^2 + 2kx + 1 \\ y = 2x - 3 \end{bmatrix} \equiv$ $x^2 + 2kx + 1 = 2x - 3$

$x^2 + 2kx - 2x + 4 = 0$

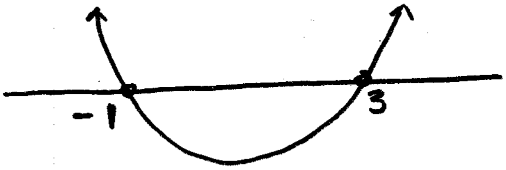
$$x^2 + (2k-2)x + 4 = 0$$

$$\begin{aligned} D &= (2k-2)^2 - (4)(4) \\ &= 4k^2 - 8k + 4 - 16 \\ &= 4k^2 - 8k - 12 \end{aligned}$$

D is a function of k,

$$\begin{aligned} D(k) &= 4k^2 - 8k - 12 \\ &= 4(k^2 - 2k - 3) \\ &= 4(k-3)(k+1) \end{aligned}$$

$D(k) = 4k^2 - 8k + 12$



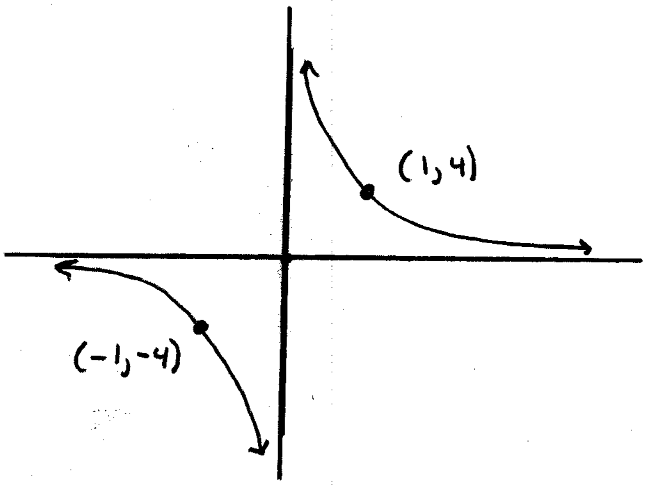
$\therefore$ $k < -1$ or $k > 3$ $\Rightarrow$ Two points shared

$k = -1$ or $k = 3$ $\Rightarrow$ one point multiplicity two

$-1 < k < 3$ $\Rightarrow$ No points shared.

P194

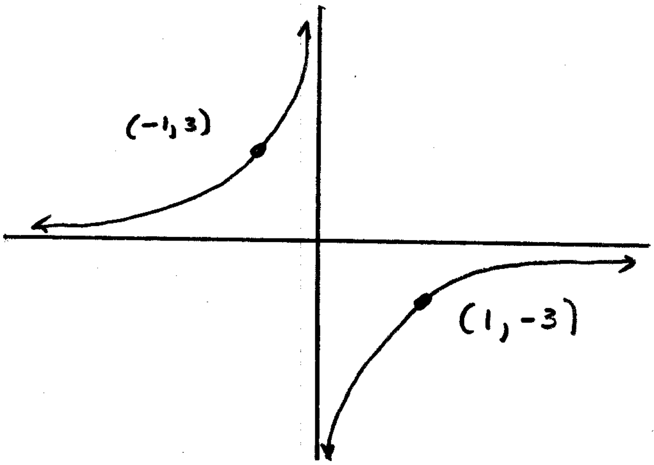
[1] $y = \frac{4}{x}$



$$\begin{bmatrix} y = \frac{4}{x} \\ y = x \end{bmatrix} \Rightarrow x = \frac{4}{x} \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$$

$\therefore$ $y = \frac{4}{x}$ intersects $y = x$ at $(-2,-2)$ and $(2,2)$.

[2] $y = -\frac{3}{x}$



$$\begin{bmatrix} y = -\frac{3}{x} \\ y = -x \end{bmatrix}$$

$$-x = -\frac{3}{x}$$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$

$\therefore$ intersections: $(-\sqrt{3}, \sqrt{3})$ and $(\sqrt{3}, -\sqrt{3})$

P195

THESE ARE correct WORKINGS FOR J10 p195 #3.1, 3.2, 3.3, 4.2

The other graphs are correct.

[3.1] $y = \frac{2}{x-3}$

assymptotes: $x=3$, $y=0$

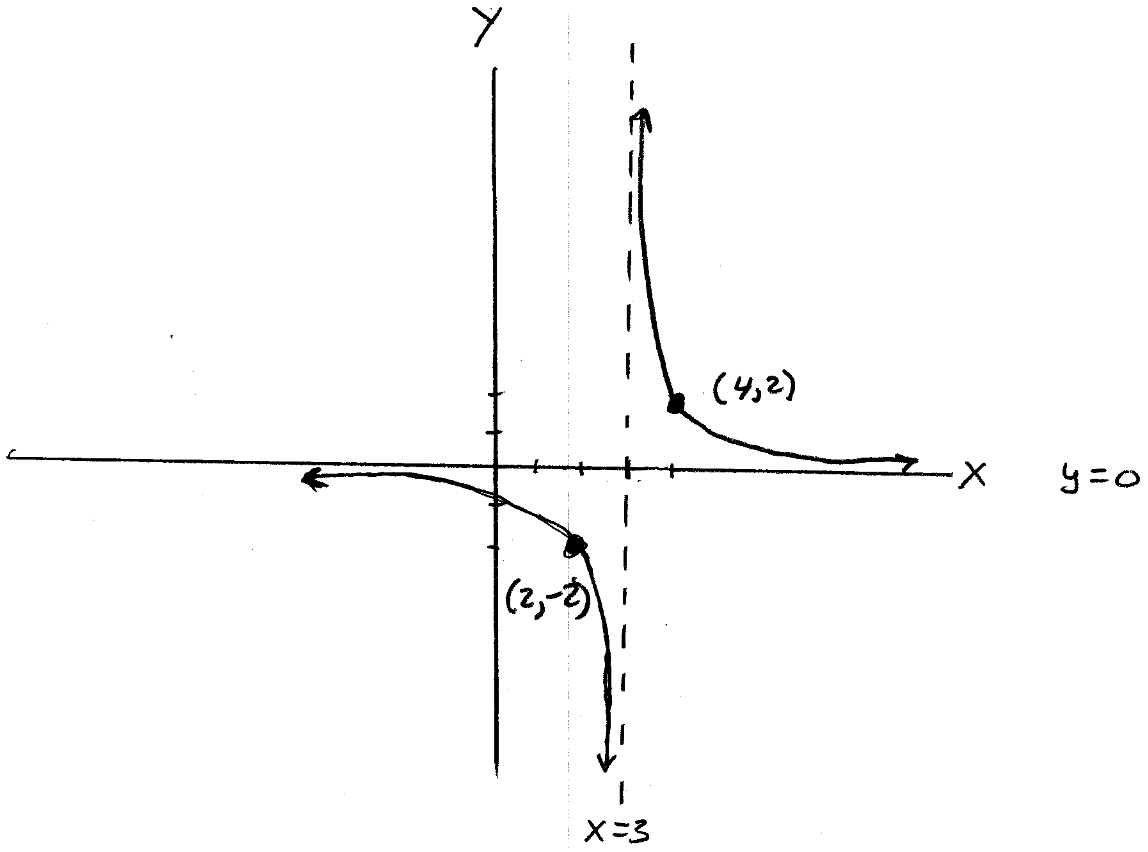
TO GRAPH: translate $y=\frac{2}{x}$ ← key idea

$(0,0) \rightarrow (3,0)$
$(1,2) \rightarrow (4,2)$
$(-1,-2) \rightarrow (2,-2)$

↑ Points on $y=\frac{2}{x}$

↑ Points on $y = \frac{2}{x-3}$

note that points (4,2), (2,-2) must satisfy $y=\frac{2}{x-3}$ could check your work this way.



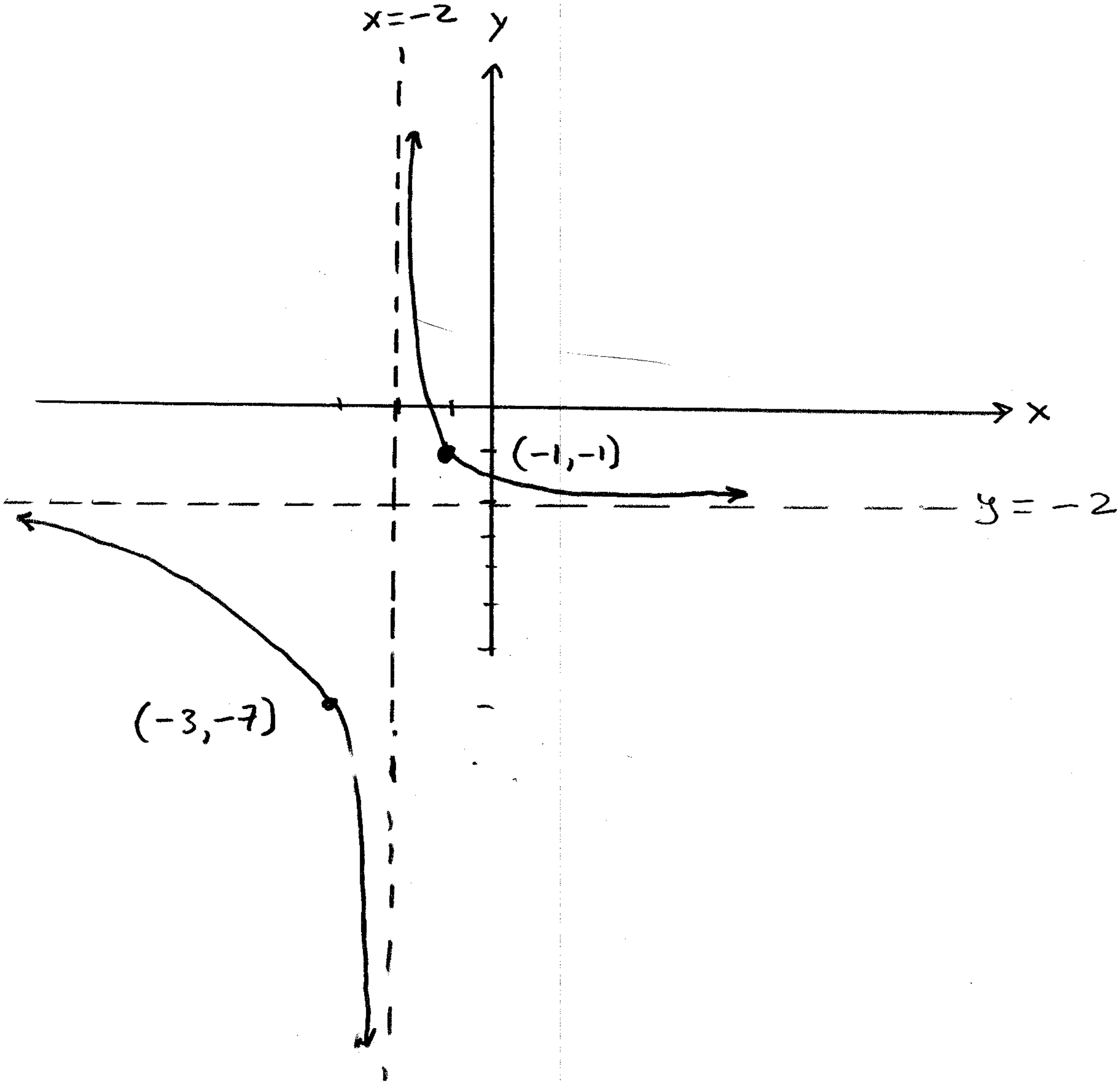
P 195, ctd

[3.2] $y = \frac{3}{x+2} - 4$

assymtotes: $x = -2,\ y = -4$

To Graph: translate $y = \frac{3}{x}$

$$y=\frac{3}{x}\left\{\begin{array}{l}(0,0)\longrightarrow(-2,-4)\\(1,3)\longrightarrow(-1,-1)\\(-1,-3)\longrightarrow(-3,-7)\end{array}\right]\ y=\frac{3}{x+2}-4$$



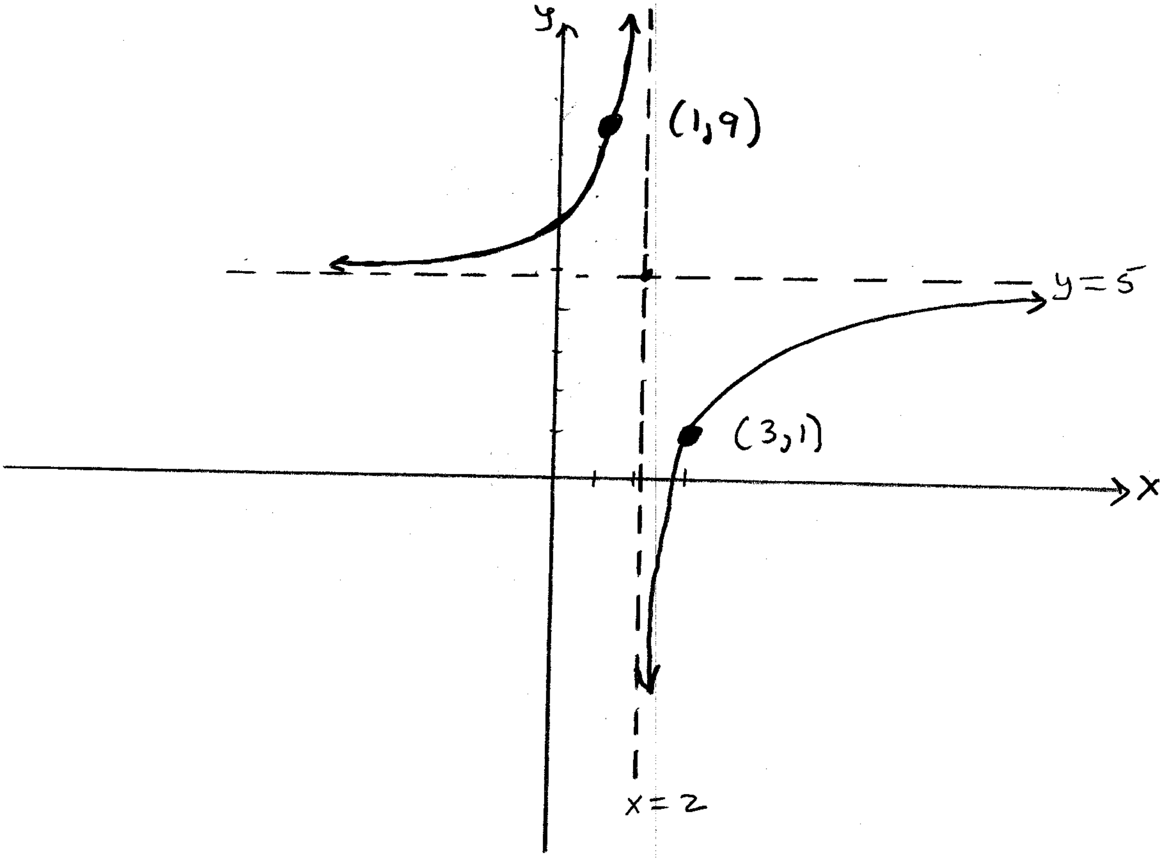
P 195, ctd

[3.3] $y = -\frac{4}{x-2} + 5$

assymptotes: $x = 2, y = 5$

TO Graph, translate $y = \frac{-4}{x}$

$$y = \frac{-4}{x} \left[\begin{array}{l} (0,0) \longrightarrow (2,5) \\ (1,-4) \longrightarrow (3,1) \\ (-1,4) \longrightarrow (1,9) \end{array} \right] y = \frac{-4}{x-2} + 5$$

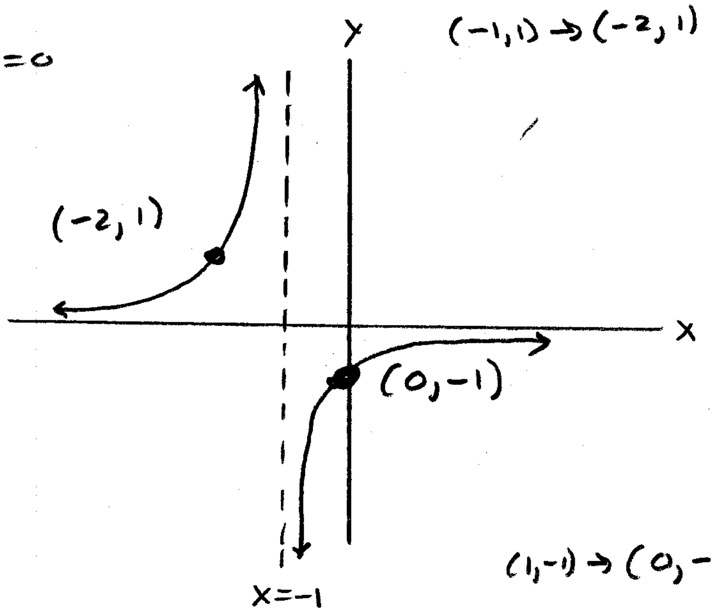


P195, ctd

[4.1] $y = \frac{-1}{x+1}$

ASSYMP: $x = -1$, $y = 0$

QUADS II, IV



(-1,1) → (-2, 1)

(1,-1) → (0,-1

P 195, ctd

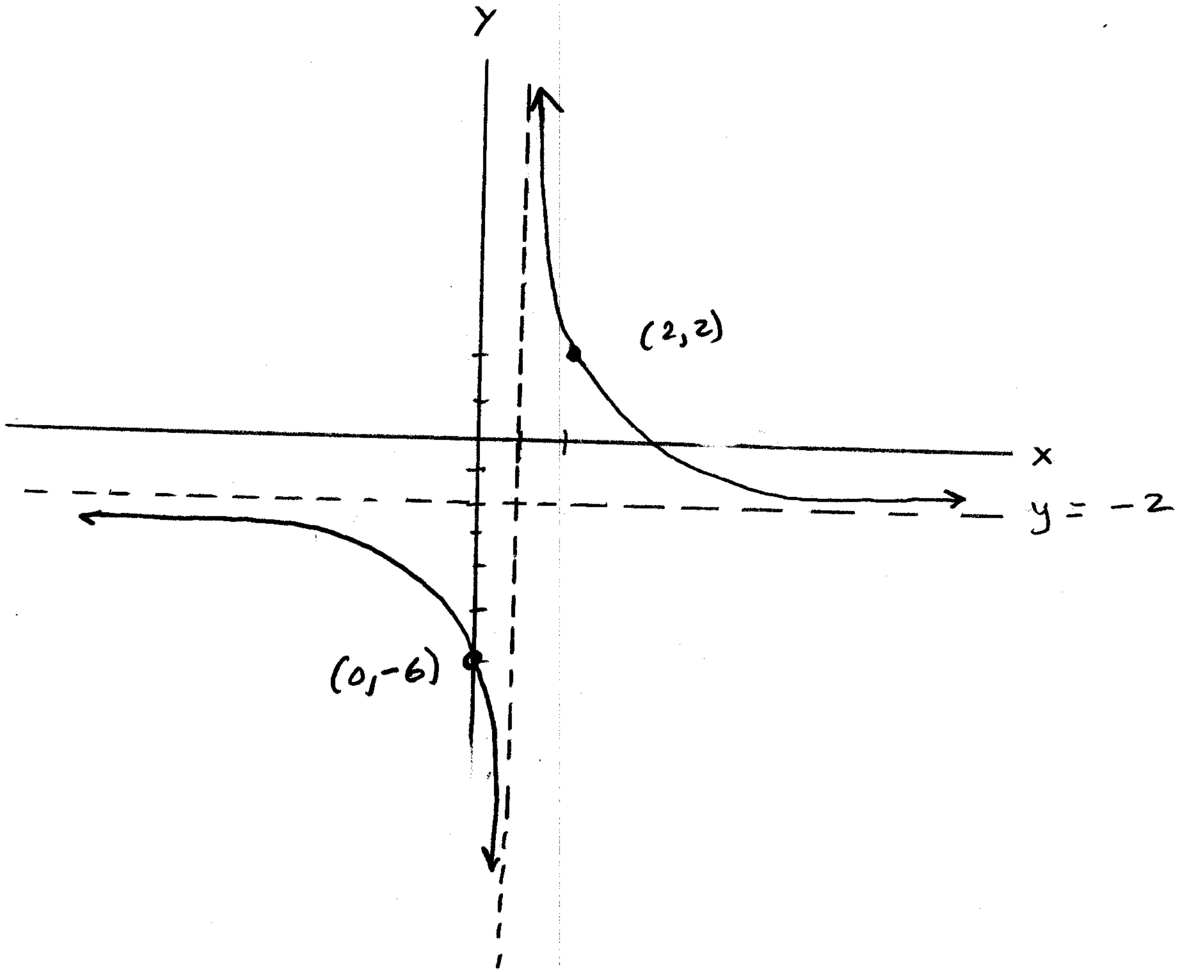
[4.2] $(x-1)(y+2)=4$

$$y+2=\frac{4}{x-1}$$

$$y=\frac{4}{x-1}-2$$

assymptotes $x=1,\ y=-2$

$$y=\frac{4}{x}\left\{\begin{array}{l}(0,0)\longrightarrow(1,-2)\\(1,4)\longrightarrow(2,2)\\(-1,-4)\longrightarrow(0,-6)\end{array}\right\}\ y=\frac{4}{x-1}-2$$



p 196

[5.1]

$$y = \frac{3x}{x-1}$$

$$= \frac{3}{x-1} + 3$$

$$\begin{array}{rl} & \quad 3 \\ x-1 & \overline{)\,3x} \\ & \underline{\;3x-3\;} \\ & \qquad 3 \end{array}$$

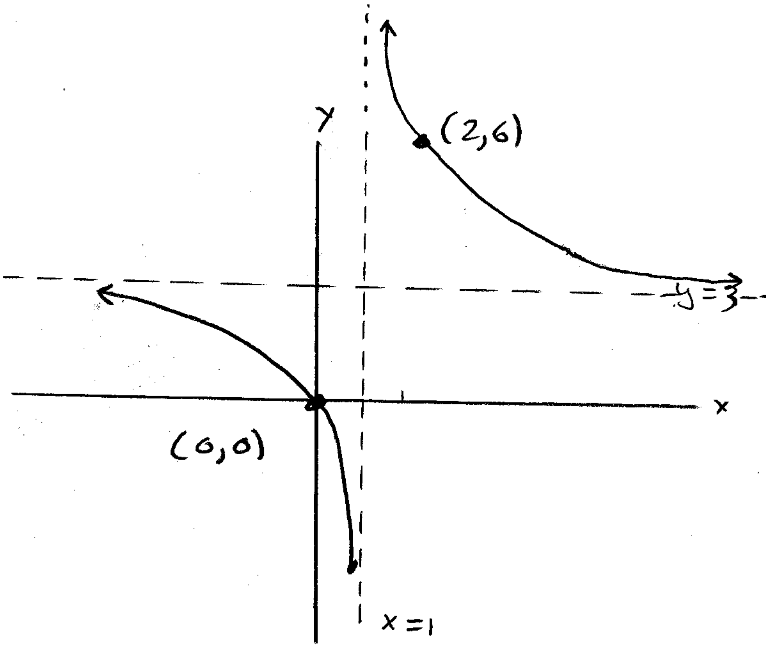
Check

$$\frac{3}{x-1} + 3 = \frac{3 + 3x - 3}{x-1} \quad \checkmark$$

Assymptotes : $x = 1$, $y = 3$

$(1,3) \rightarrow (2,6)$

$(-1,-3) \rightarrow (0,0)$



[5.2]

$$y = \frac{2x-7}{x-2}$$

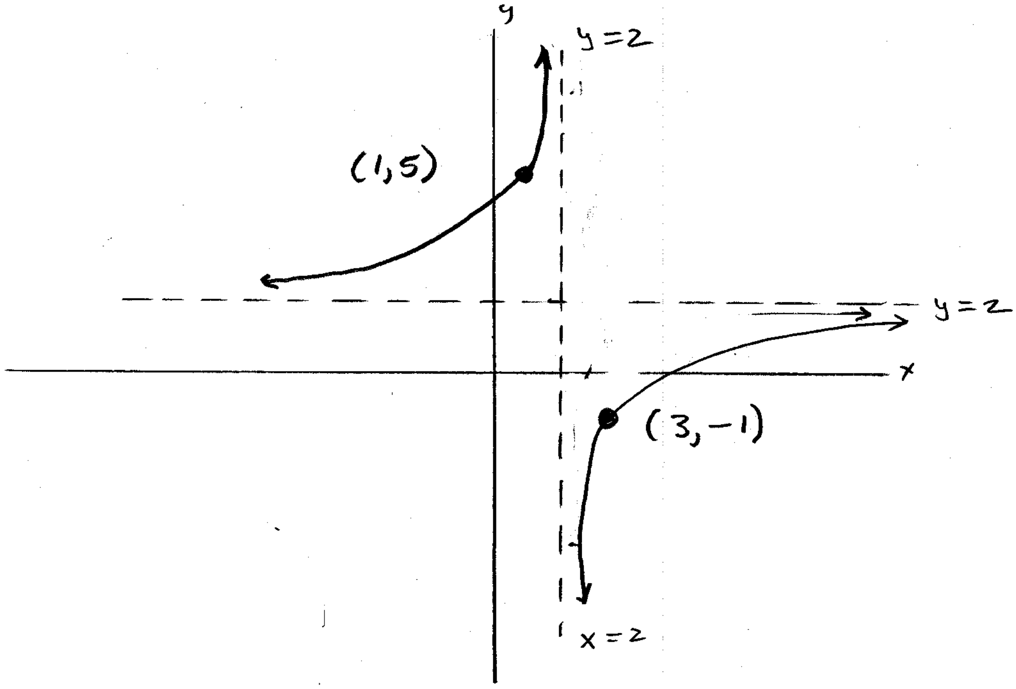
$$\begin{array}{rl} & \quad 2 \\ x-2 & \overline{)\,2x-7} \\ & \underline{\;2x-4\;} \\ & \qquad -3 \end{array}$$

$$y = 2 + \frac{-3}{x-2}$$

assymp: $x = 2$, $y = 2$

$(1,-3) \longrightarrow (3,-1)$

$(-1,3) \longrightarrow (1,5)$



P 196, ctd

[5.3]

$$y = \frac{1-2x}{x+2} = \frac{-2x+1}{x+2}$$

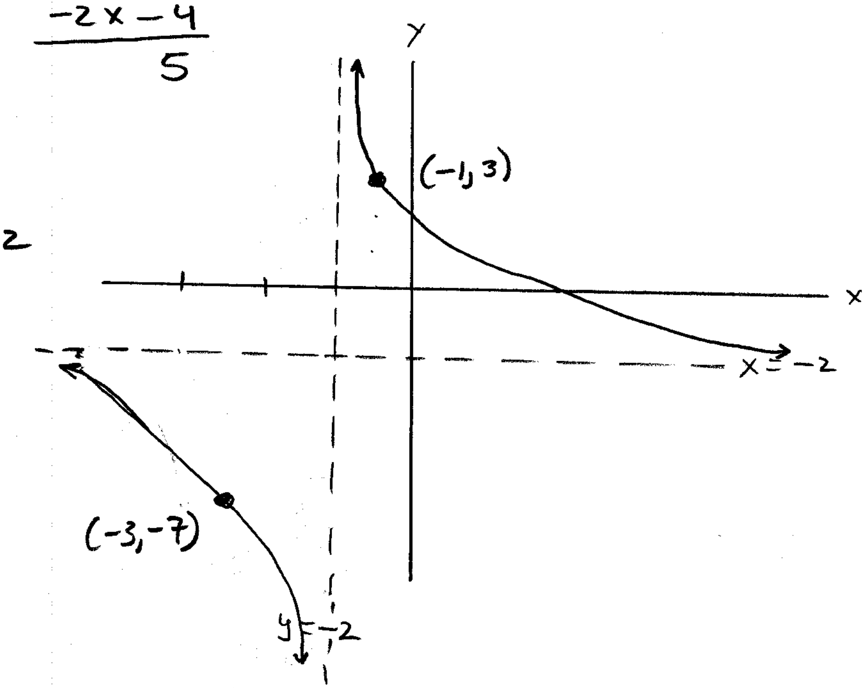
$$\begin{array}{r|l} & -2 \\ \hline x+2 & -2x+1 \\ & -2x-4 \\ \hline & \quad 5 \end{array}$$

$$y = -2 + \frac{5}{x+2}$$

assymp: $x = -2,\ y = -2$

$(1, 5) \rightarrow (-1, 3)$

$(-1, -5) \rightarrow (-3, -7)$



[5.4]

$$y = \frac{x-1}{3-x}$$

$$\begin{array}{r|l} & -1 \\ \hline 3-x & x-1 \\ & +x-3 \\ \hline & \quad 2 \end{array}$$

$$y = -1 + \frac{2}{3-x}$$

assymp: $x=3,\ y=-1$

$(1, 2) \rightarrow (4, 1)$

$(-1, -2) \rightarrow (2, -3)$